

# Anomalies of Transport in Reflectionally Noninvariant Turbulence

*A.V.Chechkin, V.V.Yanovsky,*

Laboratory for Turbulence Research

Institute for Single Crystals National Acad.Sci.Ukraine,

Lenin ave. 60, Kharkov 310001, Ukraine

and

*A.V.Tur*

Observatoire Midi–Pyrenees, 14, av. Edouard–Belin,

F31400 Toulouse, France

## Abstract

We consider the transport of passive admixture in locally homogeneous isotropic reflectionally noninvariant turbulence of incompressible fluid. It is shown that anomalous convective flow appears which direction does not coincide with that of a mean flow.

## 1 Introduction

There are many papers devoted to the study of passive admixture transport in a turbulent medium (see, e.g., [1, 2, 3]). The interest to this problem is explained by its importance and deep theoretical tasks connected with its solution. To solve the problem it is necessary to advance in understanding the problems connected with the turbulence.

The theory of the transport in a homogeneous and isotropic turbulence has been developed best of all. This is explained by the success in understanding this type of the turbulence. One of the first results, which is already classical now, is Richardson law [4] (4/3 power), which determines the relative diffusion coefficient in Kolmogorov turbulence [5]. The appearance of new ideas, connected with fractal nature of dissipation zones [6–8] leads to the generalization of the Richardson law [9,10].

Recently, the methods of the field theory were intensively applied to the calculation of anomalous transport coefficients in homogeneous isotropic turbulence. The renormalization group method seems to be most effective [11, 12].

In general, turbulent transport is determined by two principally different effects. First of all, this is a diffusion extension of an admixture "cloud". Such an extension is due to the appearance of the diffusion flow  $\vec{J}_D = -D_T \nabla \langle C \rangle$ , where  $\langle C \rangle$  is the mean concentration of the passive admixture. The coefficient  $D_T$  of concentration gradient (in general case it is a tensor) is the turbulent diffusion coefficient. It determines quantitative characteristics of the extension of the passive admixture cloud.

The transport of the cloud as a whole with changing its center position is determined by the convective flow  $\vec{J}_e = \vec{V} \langle C \rangle$ , where  $\vec{V}$  is the transport velocity which determines the value and the direction of the convective flow of passive admixture.

In a homogeneous isotropic turbulence there is no convective flow, and the evolution of a passive scalar is determined by the diffusion flow only. A lot of papers is devoted to the calculation of turbulent diffusion coefficients just for this case (see, e.g., [2, 3, 13–15]). The case of a locally homogeneous isotropic turbulence with  $\langle \vec{V} \rangle = \text{const}$ , actually, is reduced to the previous case when one passes to a reference frame moving with the velocity  $\langle \vec{V} \rangle$ . In this case the convective flow is trivial,  $\vec{J}_e = \langle \vec{V} \rangle \langle C \rangle$ , and the transport of the cloud as whole is determined by the value and the direction of  $\langle \vec{V} \rangle$ .

More interesting and nontrivial effects in the convective transport should be expected, at least, in the case of a weak spatial dependence of  $\langle \vec{V} \rangle$ . In this case there is no possibility to pass to single reference frame moving with the velocity of the medium.

In our paper the main attention is devoted to nontrivial effects which determine the convective flow of a passive admixture "cloud". Nevertheless, diffusion flows are also studied.

As it is shown in our paper, a weak dependence of  $\langle \vec{V} \rangle$  on coordinates is not sufficient for changing the convective flow. It is necessary to take into account more delicate properties of the turbulence. The property of helicity plays the principal role in the appearance of nontrivial effects. The helical turbulence is one more type of the turbulence. This type is characterized by the pseudoscalar  $\langle \vec{V} \text{curl} \vec{V} \rangle$  (helicity) which differs from zero. The reflected symmetry is violated in the helical turbulence, this property being not restored in the developed turbulence [16]. The reason for a substantial role of the helicity is due to its topological nature (see, e.g., [17–19]). The non-zero value of the helicity implies the linking of vortex lines of the velocity field, that is, a nontrivial topology of this field.

In the presence of helical turbulence a substantial reconstruction of classical instabilities, e.g., convective instability [20], takes place. Probably, it is convective instability

which the birth of typhoon is connected with [21].

In this paper the peculiarities of the passive admixture transport arising in reflectionally noninvariant turbulence are discussed. The main attention is devoted to the derivation of the equation for the average concentration and to the qualitative effects due to helicity. It can be understood from the symmetric considerations, that in order for new effects to be realized the additional factors besides helicity should exist, for example, the inhomogeneous mean velocity of the medium. In some sense, this fact implies that we approach closer to the problems of a more realistic turbulence.

Therefore, in this paper the transport in a turbulent medium with the mean velocity depending on coordinates is considered. Turbulent fluctuations are assumed reflectionally noninvariant and small-scale (comparatively to the scale of the mean flow).

With the use of multiscale formalism the equation is obtained which describes the evolution of the average concentration of the passive admixture in this turbulent medium. In this equation there are two anomalous terms describing turbulent diffusion and turbulent convective transport, respectively. The turbulent diffusion coefficient and the anomalous convective velocity depend on the mean velocity. The coefficient of turbulent diffusion does not depend on helicity. The isotropic reflectionally invariant turbulence leads only to the enhancing the diffusion flows. The role of the reflection noninvariance of the turbulence is more radical, because new convective flows of admixture are formed in helical turbulence. It is worthwhile note that the direction of new convective flows does not coincide with the direction of the mean velocity. In other words, the influence of nontrivial topology of a turbulent velocity field manifests itself in changing the direction and the value of the convective transport of a passive admixture cloud. This fact is discussed in the paper in details.

The existence of such flows can be principal for understandings the admixture transport in astrophysical and geophysical phenomena.

## 2 Average equation for the passive admixture transport

Following conventional statement of problems concerning passive admixture transport in a given turbulent field (so-called kinematic statement [18]), we take the transport equation for a passive admixture as initial one in our approach:

$$\frac{\partial C}{\partial t} + \vec{V} \nabla C = D_M \nabla C, \quad (1)$$

where  $C(\vec{x}, t)$  is the density field of the passive admixture,  $D_M$  is the molecular diffusivity,  $\vec{V}(\vec{x}, t)$  is the incompressible velocity field,  $\operatorname{div}\vec{V} = 0$ , which can be represented in the form

$$\vec{V} = \langle \vec{V} \rangle + \delta\vec{V} , \quad (2)$$

where  $\langle \vec{V} \rangle$  is the velocity of the regular flow which is assumed to be a function of  $\vec{x}$ ,  $\delta\vec{V}(\vec{x}, t)$  is the turbulent velocity field, symbol  $\langle \cdot \rangle$  implies averaging over realizations of the stochastic velocity field. Equation (1) can be written in a more fundamental form as a continuity equation :

$$\frac{\partial C}{\partial t} + \operatorname{div}\vec{J} = 0 , \quad (3)$$

where

$$\vec{J} = \vec{V}C - D_M \nabla C \quad (4)$$

is the vector of the admixture flow.

The form of transport equation (3) and (4) is the most general and physically motivated. This form allows us to differentiate the terms according to their physical meaning in a correct way [22]. The first term in Eq.(4) describes the convective transport of an admixture, that is, an initial distribution of the admixture  $C(\vec{x}, t)$  is carried with the speed  $\vec{V}(\vec{x}, t)$ , while the second term in Eq.(4) describes the extension of the initial distribution due to molecular diffusion.

Our task is to derive the equation for the average concentration  $\langle C \rangle$ . When doing it we use natural physical assumption that characteristic space and time scales of turbulent fluctuations are much smaller then the characteristic space and time scales of the average quantities. This assumption allows us to use the asymptotic method of multiscale expansion [23] in order to derive the average diffusion equation. We introduce "slow" variables  $\vec{X}, T_i, i = 1, 2, \dots$  which characterize average quantities, together with "fast" variables  $\vec{x}, t$ , which characterize small-scale turbulent fluctuations.  $\vec{x}, t$  are connected with  $\vec{X}, T_i$  as

$$\vec{X} = \varepsilon \vec{x}; \quad T_i = \varepsilon^i t , \quad (5)$$

and

$$\begin{aligned} \frac{\partial}{\partial \vec{x}} &\rightarrow \frac{\partial}{\partial \vec{x}} + \varepsilon \frac{\partial}{\partial \vec{X}} \\ \frac{\partial}{\partial t} &\rightarrow \frac{\partial}{\partial t} + \varepsilon \frac{\partial}{\partial T_1} + \varepsilon^2 \frac{\partial}{\partial T_2} + \dots . \end{aligned}$$

$\varepsilon$  is a small parameter characterizing the ratio of scales of turbulent fluctuations and the average flow. The introduction of the single space scale and the set of time scales implies that we restrict our consideration by the processes on the space scale which are of

the order of that of the mean flow, nevertheless, we want to follow the evolution of these processes for a "sufficiently long" time.

In multiscale formalism the solution of Eq.(1) (or Eq.(3)) can be found in the form of a function depending on fast and slow variables, which are assumed as independent. The solution is found in the form of asymptotic expansion in powers of  $\varepsilon$  :

$$C = \langle C \rangle + \varepsilon C_1 + \varepsilon^2 C_2 + \dots , \quad (6)$$

where  $\langle C \rangle$  is the average concentration which depends only on slow variables  $\vec{X}, T_i; C_i, i = 1, 2, \dots$  depend on both fast and slow variables. The next two equations in the hierarchy are

$$O(\varepsilon) : \frac{\partial C_1}{\partial t} + \frac{\partial \langle C \rangle}{\partial T_1} + \langle \vec{V} \rangle \frac{\partial C_1}{\partial \vec{x}} + \delta \vec{V} \frac{\partial \langle C \rangle}{\partial \vec{X}} + \langle \vec{V} \rangle \frac{\partial \langle C \rangle}{\partial \vec{X}} + \delta \vec{V} \frac{\partial C_1}{\partial \vec{x}} = D_M \frac{\partial^2 C_1}{\partial \vec{x}^2}, \quad (7)$$

$$\begin{aligned} O(\varepsilon^2) : & \frac{\partial C_2}{\partial t} + \frac{\partial C_1}{\partial T_2} + \frac{\partial \langle C \rangle}{\partial T_1} + \langle \vec{V} \rangle \frac{\partial C_2}{\partial \vec{x}} + \delta \vec{V} \frac{\partial c_2}{\partial \vec{x}} + \langle \vec{V} \rangle \frac{\partial C_1}{\partial \vec{X}} = \\ & = 2D_M \frac{\partial^2 C_1}{\partial \vec{x} \partial \vec{X}} + D_M \frac{\partial^2 \langle \vec{V} \rangle}{\partial \vec{X}^2} + D_M \frac{\partial^2 C_2}{\partial \vec{x}^2} . \end{aligned} \quad (8)$$

For the mean concentration at zeroth order approximation the evolution equations at slow times  $T_1, T_2$  are obtained from Eqs. (7), (8) as the solvability conditions (secular equation) :

$$\frac{\partial \langle C \rangle}{\partial T_1} + \langle \vec{V} \rangle \frac{\partial \langle C \rangle}{\partial \vec{X}} = 0 . \quad (9)$$

$$\frac{\partial \langle C \rangle}{\partial T_2} = D_M \frac{\partial^2 \langle C \rangle}{\partial \vec{X}^2} - \langle \vec{V} \frac{\partial C_1}{\partial \vec{X}} \rangle . \quad (10)$$

The equation for  $C_1$  obtained by subtracting Eq.(9) from Eq.(7). Then, we solve the equation for  $C_1$  formally with using the Green's function and insert the solution into Eq.(10). Thus, we get

$$\frac{\partial \langle C \rangle}{\partial T_2} = D_M \frac{\partial^2 \langle C \rangle}{\partial \vec{X}^2} + \frac{\partial}{\partial X_1} \Pi_{ij} \frac{\partial}{\partial X_j} \langle C \rangle . \quad (11)$$

where

$$\Pi_{ij} = \int dt' \int d\vec{x}' \langle G \delta V_i(\vec{x}, t) \delta V_j(\vec{x}', t') \rangle , \quad (12)$$

and the Green's function obeys an equation

$$\left[ \frac{\partial}{\partial t} + \left( \langle \vec{V} \rangle + \delta \vec{V} \right) \frac{\partial}{\partial \vec{x}} - D_M \frac{\partial^2}{\partial \vec{x}^2} \right] G = \delta(\vec{x} - \vec{x}') \delta(t - t') . \quad (13)$$

Eq.(9) describes the evolution of  $\langle C \rangle$  at the times of the order of  $\varepsilon^{-1}$ . It illustrates a rather obvious fact that, if the mean velocity exists in turbulent flow, then the main effect is the transport of an admixture cloud as a whole with this velocity. Eq.(11) describes

the evolution of  $\langle C \rangle$  at the times of the order of  $\varepsilon^{-2}$ . Thus, turbulent flows are essential at the times of  $\varepsilon^{-2}$ .

In general case the tensor  $\Pi_{ij}$  can be decomposed as sum of symmetric  $\Pi_{ij}^S \equiv D_{ij}$  and antisymmetric  $\Pi_{ij}^A$  parts. Then, Eq.(11) can be written as

$$\frac{\partial \langle C \rangle}{\partial T_2} + \frac{\partial}{\partial X_i} \left[ W_i \langle C \rangle - D_{ij} \frac{\partial \langle C \rangle}{\partial X_j} - D_M \frac{\partial \langle C \rangle}{\partial X_j} \right] = 0 \quad (14)$$

where  $W_i = \partial \Pi_{ij}^A / \partial X_j$ , and the convective and the diffusion flows are separated.

Thus, we have demonstrated that the average transport equation at times of the order of  $\varepsilon^{-2}$  contains two main turbulent effects, namely, the turbulent diffusion flow  $-D_{ij} \partial \langle C \rangle / \partial X_j$  and the anomalous convective flow  $\vec{W} \langle C \rangle$ .

Turbulent diffusion leads to the extension of an admixture cloud, the characteristic size of the cloud grows as a square root of the slow time. The convective flow  $\vec{W} \langle C \rangle$  leads to the transport of the cloud as whole, the deviation of the coordinate of the cloud center grows proportionally to time. This circumstance leads to the fact that at sufficiently large times the effect of the anomalous convective flow will prevail over the effect of the turbulent diffusion. In other words, the characteristic size of the diffusive extension becomes smaller than the deviation of the cloud center due to the anomalous convective flow.

### 3 Generalized helicity and anomalous convective flow

In this Section we show that the convective flow is non-zero for locally homogeneous isotropic reflectionally noninvariant turbulence.

Usually in a kinematic statement of problems concerning passive admixture transport one does not use an explicit form of velocity correlation tensor [18]. It is possible due to simple representation of the tensors, which are invariant to certain space transformations [3,4] (we mention alpha-effect in magnetic dynamo theory as a classic example). Let us start with the case of homogeneous isotropic reflectionally noninvariant turbulence  $\delta \vec{V}(\vec{x}, t)$ . Due to the invariance of the statistical characteristics of the turbulence under dilatations and rotations, the correlation tensor  $\langle \delta V_i \delta V_j \rangle$  in general case is a linear combination of elementary tensors  $\delta_{ij}, r_i r_j (\vec{r} = \vec{x} - \vec{x}')$  and a pseudotensor  $\varepsilon_{ijl} r_l$  with the coefficients depending on  $|\vec{r}|$  (it is also invariant under transformations mentioned above). It is important to note that the coefficients at  $\delta_{ij}$  and  $r_i r_j$  are scalars while the coefficient at  $\varepsilon_{ijl} r_l$  is a pseudoscalar. This fact is a consequence of the invariance of  $\langle \delta V_i \delta V_j \rangle$  also under the reflections of coordinate axes. Nevertheless, in spite of the fact that  $\langle \delta V_i \delta V_j \rangle$  is invariant under the reflections, the pseudoscalar in it is also the correlation moment of

the velocity field. In this case it coincides with the helicity. This moment is noninvariant under the reflections. The turbulence with nonzero correlation moments noninvariant under the axes reflection can be naturally called reflectionally noninvariant. As a matter of fact, this property extracting the pseudoscalar coefficient can be regarded as another definition of the helicity.

Further on we shall be interested in the explicit form of the tensor  $\Pi_{ij}$ . Let us consider the tensor  $\langle G\delta V_i \delta V_j \rangle$  in  $\Pi_{ij}$ . The stochastic field  $\delta \vec{V}$  is homogeneous and isotropic as before. However, the Green's function  $G$  depends on the vector  $\langle \vec{V} \rangle$ , therefore, the general form of the tensor  $\langle G\delta V_i \delta V_j \rangle$  is more complex than the tensors discussed previously. This case is close to that of axisymmetric turbulence [24]. In other words, the number of elementary tensors which determine the general form of  $\langle G\delta V_i \delta V_j \rangle$  grows. Writing all possible elementary tensors and pseudotensors, namely,  $\delta_{ij}, r_i r_j, r_l \langle V_j \rangle, \langle V_i \rangle \langle V_j \rangle, \varepsilon_{ijl} r_l, \varepsilon_{ijl} \langle V_l \rangle$  etc., see [24], with coefficients depending on  $|\vec{r}|, |\langle \vec{V} \rangle|$  and  $\vec{r} \cdot \langle \vec{V} \rangle$  (invariant combinations of the vectors  $\vec{r}$  and  $\langle \vec{V} \rangle$ ) we get a general form of the tensor  $\langle G\delta V_i \delta V_j \rangle$ . After integration over space coordinates we get an expression for the tensor  $\Pi_{ij}$ :

$$\Pi_{ij} = \int_0^\infty d\tau \{ A_1 \delta_{ij} + A_2 \langle V_i \rangle \langle V_j \rangle + H \varepsilon_{ijl} \langle V_l \rangle \} , \quad (15)$$

where  $A_1, A_2$  are scalar functions of  $|\langle \vec{V} \rangle|, \tau$ , while  $H$  is a pseudoscalar function. The terms with  $A_1, A_2$  are symmetric on indices  $i, j$ , and they determine the turbulent diffusion, that is, the term  $D_{ij}$  in Eq.(14). If the mean velocity is equal to zero, then only the term with  $A_1$  retains in Eq.(15). If non-zero mean velocity exists, then the term with  $A_2$  appears in the turbulent diffusion coefficient. This term describes the influence of the mean velocity on the diffusion processes. However, the last term in Eq.(15) leads to more radical consequences. This term is antisymmetric on indices  $i, j$  and, as it follows from the consideration above, leads to an anomalous convective flow:

$$W_1 = \frac{\partial D_{ij}^A}{\partial X_j} = G_1 \left( \nabla |\langle \vec{V} \rangle| \times \langle \vec{V} \rangle \right)_i + G_2 \text{curl}_i \langle \vec{V} \rangle , \quad (16)$$

where

$$G_1(|\langle \vec{V} \rangle|) = \int_0^\infty d\tau \frac{\partial}{\partial |\langle \vec{V} \rangle|} H(|\langle \vec{V} \rangle|, \tau) , \quad (17)$$

$$G_2(|\langle \vec{V} \rangle|) = \int_0^\infty d\tau H(|\langle \vec{V} \rangle|, \tau) . \quad (18)$$

Let us turn to the physical meaning of the pseudoscalar  $H$ . The nonzero value of  $H$  implies the existence of some correlation moments of the turbulent field  $\delta \vec{V}$  such that these moments change sign under the reflection of coordinate axes. Just these moments

form the quantity  $H$ . In this sense  $H$  is a measure of reflection noninvariance of the turbulence considered. Therefore, we call it generalized helicity because the "ordinary" helicity possesses the same property. Moreover, if we expand the Green's function in the orders of  $\delta\vec{V}$ , then in the zero-order approximation  $H$  is proportional to the ordinary helicity density.

Thus, the effect of anomalous convective transport appears in reflectionally noninvariant turbulence, and the value of convective transport is determined by the generalized helicity which, in part, is non-zero for the turbulence possessing an ordinary helicity.

An explicit expressions for the turbulent diffusion and the anomalous convective flow can be obtained in various limit cases by solving the equation (13) for the Green's function by different methods, e.g., the Green's function of zeroth order approximation on fluctuations serving as bare one; with using the assumption of the Gaussian character of the velocity fluctuations and the Furutzu–Novikov formula; with the help of an expansion in small Peclé numbers, etc. The simplest expressions can be obtained by neglecting velocity fluctuations and molecular diffusion in Eq.(13) (it corresponds to the zeroth order approximation of the diagram technique). Then the Green's function takes the form

$$G = \theta(t - t')\delta(x - x' - \langle \vec{V} \rangle(t - t')) \quad (19)$$

and  $\Pi_{ij}$

$$\Pi_{ij} = \int_0^t \delta\tau \langle \delta V_i \delta V_j \rangle_{\langle \vec{V} \rangle_{\tau, \tau}} \quad (20)$$

Eq.(11) with  $\Pi_{ij}$  given by Eq.(20) has been obtained previously [26] with using assumption that the turbulent velocity is a telegraph stochastic process. Eqs.(19), (20) describe turbulent diffusion in homogeneous isotropic nad anisotropic turbulence as well.

Inserting two-point correlation tensor of homogeneous isotropic turbulence [3,4] into Eq.(20) one can easily see that the generalized helicity is reduced to an ordinary one, whereas in Eq.(16)-(18)

$$H(|\langle \vec{V} \rangle|, \tau) \rightarrow g(|\langle \vec{V} \rangle| \tau, \tau)$$

where

$$g(0, 0) = \frac{1}{6} \langle \delta \vec{V}(\vec{x}, t) \cdot \text{curl} \delta \vec{V}(\vec{x}, t) \rangle$$

coincides (up to numerical factor) with the "ordinary" helicity density.

The effect considered is the most interesting in the cases when the direction of the anomalous convective flow is orthogonal to the direction of the flow  $\langle \vec{V} \rangle \langle C \rangle$ :

1. The velocity gradient is perpendicular to the velocity direction. Such a situation is rather frequent; the boundary layer of the atmosphere can be mentioned, when the

wind velocity is parallel to the surface while the gradient of the velocity is perpendicular to the surface. Then, the anomalous convective flow is directed "sideways", that is, along the third coordinate parallel to the surface. It is interesting to note the analogy of this effect and the effect previously described by us [25] : a particle beam with an inhomogeneous velocity profile in an external stochastic helical field generates a new particle flow in the transverse direction.

Therefore, in a homogeneous isotropic reflectionally noninvariant turbulence with the gradient of the mean velocity the drift of an admixture in the direction perpendicular to the wind direction arises along with the diffusive extension.

2. The mean flow  $\langle \vec{V} \rangle$  is a large-scale plane vortex. Such vortices appear, for example, when a wind flows around an obstacle. In this case the anomalous convective flow is directed away from the surface or towards it in accordance with the sign of the helicity density. Therefore, in a laminar plane vortex flow the mechanism of lifting or settling of an admixture exists due to the reflection noninvariance of the homogeneous isotropic turbulence.

The consideration presented above demonstrates an importance of taking into account the property of reflection noninvariance of the turbulence when studying the effects of turbulent transport. Astrophysical and geophysical phenomena with a very large range of scales can provide the existence of the effects arising due to anomalous convective processes.

## 4 Results

In the paper we study the transport of a passive admixture in a helical (reflectionally noninvariant) turbulence with a weakly inhomogeneous mean flow.

1. With the use of multiscale formalism a closed equation is obtained which describes the transport of passive admixture in such a turbulence.
2. The dependence of the turbulent diffusivity tensor on the mean flow velocity is obtained.
3. It is shown that a new effect arises in this turbulence, namely, the anomalous convective transport. This effect comes in the same order as the turbulent diffusion.

4. It is demonstrated that the direction of the anomalous convective flow does not coincide with the direction of the mean flow. Natural physical cases are presented when the direction of the anomalous flow is perpendicular to the direction of the mean flow.
5. It is proved that the violence of reflection invariance of the turbulence possessing a weakly inhomogeneous flow manifests itself just in the appearance of the anomalous convective transport.

### **Acknowledgements**

This paper was supported by International Association under the project INTAS–93–1194 and by the State Committee on Science and Technologies of Ukraine, grant No 2/278.

## References

- [1] A.S.Monin, A.M.Yaglom, Statistical Fluid Meshanism. MIT Press (Cambridge, Mass.), 1971 (1965 in Russian).
- [2] D.C.Leslie, Developments in the Theory of Turbulence. Clarendon Press (Oxford), 1973.
- [3] G.K.Batchelor, The Theory of Homogeneous Turbulence (Cambridge), 1953.
- [4] L.F.Richardson , Proc.Royal.Soc.London, **A 110** (1926) 709.
- [5] A.N.Kolmogorov, Cr.Akad.Sci. (USSR) **30** (1941) 301.
- [6] B.B.Mandelbrot, The Fractal Geometry of Nature, W.N.Freeman (NY), 1982.
- [7] U.Frisch, P.L.Sulem, M.Nelkin, Journ.Fluid Mech., **87** (1978) 719.
- [8] J.L.McCauley, Phys.Reports, **198** (1990) 226.
- [9] S.Grossman, I.Procaccia, Phys.Rev. **A 29** (1984) 1358.
- [10] H.G.E.Hentschel, I.Procaccia, Phys.Rev. **A 29** (1984)1461.
- [11] V.Yakhot, S.A.Orszag, Journ.Sci.Comput. **1** (1986) 3.
- [12] W.P.Dannevik, V.Yakhot, S.A.Orszag, Phys.Fluids, **30** (1987) 1.
- [13] R.H.Kraichnan, Phys.Fluids, **13** (1970) 22.
- [14] Ya.B.Zeldovich, Dokl.Akad.Nauk SSSR **226** (1982) 821; (Sov.Phys.Dokl. **27** (1982) 797).
- [15] U.Frisch, Lectures on Turbulence and Lattice Gas Hydrodynamics. In: Lecture Notes on Turbulence. NCAR-GTP Summer School, June 1987, p. 219.
- [16] S.S.Moiceev, R.Z.Sagdeev, A.V.Tur, G.A.Khomenko, V.V.Yanovsky, Zhurn. Eksper.Teor.Fiz. (Sov.JETP) **58** (1983) 1144.
- [17] J.J.Moreau, C.R. Akad.Sci.Paris, **252** (1961) 2810.
- [18] H.K.Moffat, Magnetic Field Generation in Electrically Conducting Fluids, Cambridge Univ.Press. (Cambridge), 1978.
- [19] A.V Tur, V.V.Yanovsky, Journ.Fluid.Mech. **248** (1993) 67.

- [20] S.S.Moiseev, P.B.Rutkevich, A.V Tur, V.V.Yanovsky, Zhurn. Eksper. Teor. Fiz. (Sov.JETP) **94** (1988) 144.
- [21] R.Z.Sagdeev, S.S.Moiseev, P.B.Rutkevich, A.V Tur, V.V.Yanovsky, In: Tropical Meteorology. Proc. 3th Int.Symp. Leningrad, Gidrometeoizdat, 1987, p. 18.
- [22] V.G.Levich, Hydrodynamics of Chemical Physics (in Russian). M., Acad. Sci. USSR Publ., 1952.
- [23] Al.H.Nayfeh, Perturbation Methods, Y.Wiley & Sons, Inc., 1973.
- [24] G.S.Chandrasekhar, Phil.Trans.Eoy.Soc. **A** **242** No 855 (1950) 557.
- [25] A.V.Chechkin, A.V Tur, V.V.Yanovsky, Physica **A** **208** (1994) 501.